

# The nature of the lightest scalar meson, its $N_c$ behaviour and semi-local duality

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One-loop unitarized Chiral Perturbation Theory (UChPT) calculations, suggest a different  $N_c$  behaviour for the  $\sigma$  or  $f_0(600)$  and  $\rho(770)$  mesons: while the  $\rho$  meson becomes narrower with  $N_c$ , as expected for a  $\bar{q}q$  meson, the  $f_0(600)$  contribution to the total cross section below 1 GeV becomes less and less important. Here we review our recent work [1] where we have shown, by means of finite energy sum rules, that a different  $N_c$  behaviour for these resonances may lead to a conflict with semi-local duality for large  $N_c$ , since local duality requires a cancellation between the  $f_0(600)$  and  $\rho(770)$  amplitudes. However, UChPT calculations also suggest a subdominant  $\bar{q}q$  component for the  $f_0(600)$  with a mass above 1 GeV and this can restore semi-local duality, as we show.

## 1 Introduction

QCD perturbative calculations are not applicable to the longstanding [2] controversy on the non- $\bar{q}q$  nature of light scalar mesons. However, the QCD  $1/N_c$  expansion [3] allows for a clear identification of a  $\bar{q}q$  resonance, since it becomes a bound state, whose width behaves  $O(1/N_c)$ , and its mass as  $O(1)$ . In addition, in this low energy region one can use Chiral Perturbation Theory (ChPT) [4,5] to obtain a model independent description of the dynamics of pions, kaons and etas, which are the pseudo-Goldstone Bosons of the QCD spontaneous breaking of Chiral Symmetry. Lately, by combining the  $1/N_c$  expansion of ChPT with dispersion theory, it has been possible to study the nature of light resonances thus generated in meson-meson scattering [7,8].

Let us recall that the ChPT Lagrangian is built as a low energy expansion respecting all QCD symmetries, and using only pseudo-Goldstone boson fields. The small masses of the three lightest quarks can be treated perturbatively and thus ChPT becomes a series in momenta and meson masses, generically  $O(p^2/\Lambda^2)$ . Apart from these masses and  $f_\pi$ —the pion decay constant, which sets the scale  $\Lambda \equiv 4\pi f_\pi$ —there are no free parameters at leading order. The chiral expansion is renormalized order by order by absorbing loop divergences in the coefficients of higher order counterterms, called low energy constants (LECs), whose values depend on the underlying QCD dynamics, and have to be determined from experiment. However, the leading  $1/N_c$  dependence of the LECs is known and model independent [5], thus allowing us to study the  $N_c$  dependence of low energy hadronic observables.

## 1.1 Unitarization and dispersion theory

Unitarity implies that, for physical values of  $s$ , partial waves  $t^{IJ}$  of definite isospin  $I$  and angular momenta  $J$  for *elastic* meson-meson scattering satisfy:

$$(1) \quad \text{Im } t^{IJ} = \sigma |t^{IJ}|^2 \quad \Rightarrow \quad \text{Im } \frac{1}{t^{IJ}} = -\sigma$$

where  $\sigma = 2p/\sqrt{s}$ , and  $p$  is the CM momentum. Note that unitarity implies that  $|t^{IJ}| \leq 1/\sigma$ , and typically elastic resonances are characterized by the saturation of this bound. However, ChPT partial waves are a low energy expansion  $t \simeq t_2 + t_4 + t_6 + \dots$ , (we will drop the  $IJ$  indices for simplicity.) where  $t_{2k} \equiv O(p/(4\pi f_\pi))^{2k}$ , and thus they cannot satisfy unitarity exactly, but just perturbatively, i.e:  $\text{Im } t_2 = 0$ ,  $\text{Im } t_4 = \sigma t_2^2$ , etc ...

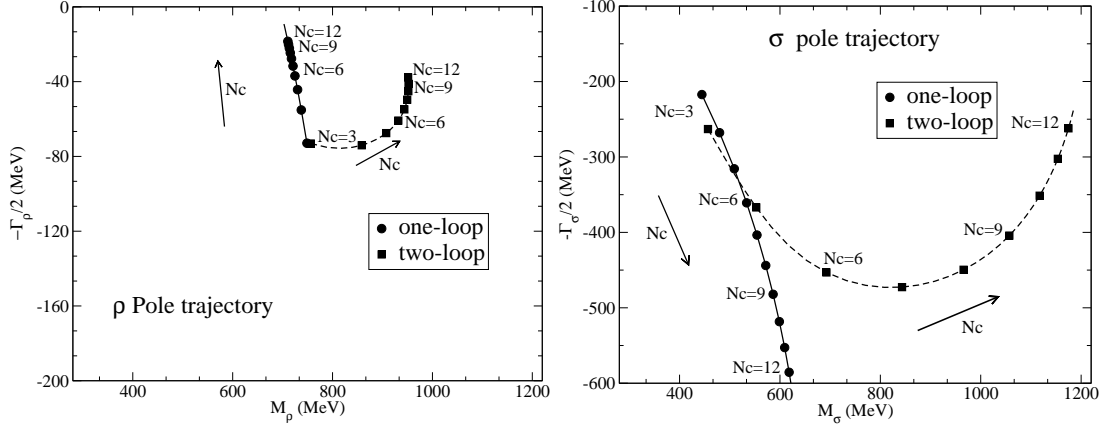
The elastic Inverse Amplitude Method (IAM) [9,10] uses ChPT to evaluate the subtraction constants and the left cut of a dispersion relation for the inverse of the partial wave. The elastic right cut is calculated exactly with Eq.(1)—thus ensuring elastic unitarity. Note that the IAM is derived only from elastic unitarity, analyticity in the form of a dispersion relation, and ChPT, which is only used at low energies. Remarkably, the IAM can be rewritten as a simple algebraic formula in terms of ChPT amplitudes, but it satisfies exact elastic unitarity, at low energies recovers the chiral expansion up to the initially given order, and reproduces meson-meson scattering data up to energies  $\sim 1$  GeV. This is done with values of the LECs which are fairly compatible with the values obtained within standard ChPT. Since it is derived from a dispersion relation, it can be analytically continued into the second Riemann sheet where, within the SU(2) ChPT formalism that we use here, we find the poles associated to the  $\rho(770)$  and  $f_0(600)$  resonances. Hence, we can study, without any *a priori* assumption, the nature of the  $\rho(770)$  and  $f_0(600)$  from first principles and QCD.

## 1.2 The $1/N_c$ expansion

For our purposes [1], the relevant observation is that the leading  $1/N_c$  behaviour of the ChPT LECs is known. Thus, in order to obtain the leading  $N_c$  behaviour of the resonances generated with the IAM, we just have to rescale  $f_\pi \rightarrow f_\pi \sqrt{N_c/3}$ , the one-loop LECs as  $l_i^r \rightarrow l_i^r N_c/3$  and the two loop ones as  $r_i \rightarrow r_i (N_c/3)^2$ .

This procedure [7] was first applied to the one-loop SU(3) ChPT amplitudes, and the result was that the light vector resonances, as for example the  $\rho(770)$ , followed the expected behaviour of  $\bar{q}q$  states remarkably well, as we show in the left panel of Fig.1. In contrast, as seen on the right panel of Fig.1 the  $f_0(600)$  behavior is at odds with that of  $\bar{q}q$  states. It follows that the dominant component of the  $f_0(600)$  ( and the other members of the lightest scalar nonet) does not have a  $\bar{q}q$  nature.

Of course, these results have some uncertainty, particularly on the renormalization scale where the  $N_c$  scale is to be applied, which was studied also in [7]. However, the general conclusions are rather robust: whereas vector mesons behave predominantly as a  $\bar{q}q$ , the



**Figure 1:** Left, the  $\rho$  meson behaves as a  $\bar{q}q$  state. Center: At  $O(p^4)$  the  $\sigma$  does not. Right: At  $O(p^6)$  emerges a subleading  $\bar{q}q$  component at 1.2 GeV

scalar ones do not, at least for  $N_c$  not far from 3, which is where the IAM can be applied (not at  $N_c \rightarrow \infty$ , see [11])

The two-loop SU(2) ChPT amplitude analysis [8] showed that when the  $\rho(770)$  is to follow its expected  $\bar{q}q$  behaviour, the  $f_0(600)$  still did not follow a  $\bar{q}q$  behavior at first, and its pole moved away from the 400-600 MeV region of the real axis. However, for  $N_c \sim 8$ , the  $f_0(600)$  started behaving as a  $\bar{q}q$ , see Fig. 1 (Right). In conclusion, the two-loop IAM confirms once again that the  $f_0(600)$  does not behave predominantly as a  $\bar{q}q$  state, but suggests the existence of a subdominant  $\bar{q}q$  component that originates at a mass of  $\simeq 1.2$  GeV, which is approximately twice that of the physical  $\sigma$  at  $N_c = 3$ . This seems to support models like [12] that have indeed suggested a non- $\bar{q}q$  nonet below 1 GeV and an additional  $\bar{q}q$  one above.

## 2 Semi-local duality

A well known feature of the real world ( $N_c = 3$ ) is that of “local duality”. At low energies the scattering amplitude is well represented by a sum of resonances (with a background), but as the energy increases the resonances (having more phase space for decay) become wider and increasingly overlap. This overlap generates a smooth Regge behaviour described by a small number of crossed channel Regge exchanges. Indeed, detailed studies of meson-baryon scattering show that the sum of resonance contributions at all energies “averages” the higher energy Regge behaviour. Thus, s-channel resonances are related to Regge exchanges in the t-channel and are “dual” to each other: one uses one or the other.

Regge exchanges are also built from  $\bar{q}q$  and multi-quark contributions. However, in the isospin 2  $\pi\pi$  scattering channel there are no  $\bar{q}q$  resonances, and so the Regge exchanges with these quantum number must involve multi-quark components. Data teach us that

even at  $N_c = 3$  these components are suppressed compared to the dominant  $\bar{q}q$  exchanges. hence, semi-local duality means that in  $\pi^+ \pi^- \rightarrow \pi^- \pi^+$ , which is an isospin 2 process, the low energy resonances must have contributions to the cross-section that “on the average” cancel. In particular, using the crossing relations the  $I=2$   $t$ -channel amplitude can be recast as a function of  $s$ -channel amplitudes:

$$(2) \quad \text{Im } A^{t2}(s, t) = \frac{1}{3} \text{Im } A^{s0}(s, t) - \frac{1}{2} \text{Im } A^{s1}(s, t) + \frac{1}{6} \text{Im } A^{s2}(s, t),$$

but since  $A^{s2}$  is repulsive and small, the strong cancellation occurs between  $A^{s0}$  and  $A^{s1}$ . However, these channels are saturated at low energies by the  $f_0(600)$  and  $\rho(770)$  resonances, respectively. Hence, semi-local duality requires the contribution of these two resonances to cancel “on average” in keeping with  $I = 2$  exchange in the  $t$ -channel. This “on the average cancellation” is properly defined via Finite Energy Sum Rules:

$$(3) \quad F(t)_n^{21} = \frac{\int_{\nu_{th}}^{\nu_{max}} d\nu \text{Im } A^{t2}(s, t) / \nu^n}{\int_{\nu_{th}}^{\nu_{max}} d\nu \text{Im } A^{t1}(s, t) / \nu^n}, \quad \nu = (s - u)/2.$$

Semi-local duality between Regge and resonance contributions teaches us that on the “average” and at least over one resonance tower, we have:

$$(4) \quad \int_{\nu_{th}}^{\nu_{max}} d\nu \nu^{-n} \text{Im } A^{t2}(s, t)_{\text{Data}} \sim \int_{\nu_{th}}^{\nu_{max}} d\nu \nu^{-n} \text{Im } A^{t2}(s, t)_{\text{Regge}}$$

where Regge amplitudes are given as usual for  $\alpha' \nu \gg 1$  by

$$(5) \quad \text{Im } A^{tI}(\nu, t) \simeq \sum_R \beta_R(t) [\alpha' \nu]^{\alpha_R(t)}$$

(see [1] for its low energy extrapolation), and where  $\alpha_R(t)$  denote the Regge trajectories with the appropriate  $t$ -channel quantum numbers,  $\beta_R(t)$  their Regge couplings and  $\alpha'$  is the universal slope of the  $q\bar{q}$  meson trajectories ( $\sim 0.9 \text{ GeV}^{-2}$ ). For the  $I = 0$  exchange the dominant trajectories are the Pomeron and the  $f_2$  with intercepts close to 1 and 0.5 respectively, while the  $I = 1$   $\rho$  exchange is degenerate with the  $f_2$ . For the exotic  $I = 2$  channel with its leading Regge exchange being a  $\rho - \rho$  cut, we expect its intercept to be much smaller than that of the  $\rho$ , and its couplings to be correspondingly smaller. Therefore using Eqs.(4) and (5), local duality implies that  $|F(t)_n^{21}| \ll 1$ .

We can now use the IAM to check the  $N_c$  dependence of  $\pi\pi$  scattering amplitudes. In particular the  $I=2$   $s$ -channel amplitude remains repulsive with  $N_c$ , and still there is no resonance exchange. Therefore semi-local duality implies that the  $t$ -channel  $I = 2$  Regge exchange should continue to be suppressed as  $N_c$  increases. Since the Regge trajectories do not depend on  $N_c$ , still  $|F(t)_n^{21}| \ll 1$  when increasing  $N_c$  due to a strong cancellation between the  $\rho(770)$  and the  $f_0(600)$  which would not occur if the  $f_0(600)$  disappeared completely from the spectrum.

$\nu_{max}$	400 GeV <sup>2</sup>	2.5 GeV <sup>2</sup>	2 GeV <sup>2</sup>	1 GeV <sup>2</sup>
$F_1^{21}$	$0.021 \pm 0.016$	$0.180 \pm 0.066$	$0.199 \pm 0.089$	$-0.320 \pm 0.007$
$F_2^{21}$	$0.057 \pm 0.024$	$0.068 \pm 0.024$	$0.063 \pm 0.025$	$-0.115 \pm 0.013$
$F_3^{21}$	$0.249 \pm 0.021$	$0.257 \pm 0.022$	$0.259 \pm 0.022$	$0.221 \pm 0.021$

**Table 1:** Values of the ratio  $F_n^{21}$  using the KPY parametrization and different cutoffs. All  $F_n^{21}$  ratios for a 20 GeV cutoff turn out very small, of the order 1:50 or 1:15. However, we see that they are only 1:4 or 1:5 when  $s_{max}$  is still  $\sim 2$  GeV<sup>2</sup>

$\nu_{max}=1$ GeV <sup>2</sup>			
	KPY08	1 loop UChPT	2 loop UChPT
$F_1^{21}$	$-0.350 \pm 0.083$	$-0.355 \pm 0.061$	$-0.351$
$F_2^{21}$	$-0.131 \pm 0.042$	$-0.157 \pm 0.097$	$-0.172$
$F_3^{21}$	$0.215 \pm 0.027$	$0.175 \pm 0.138$	$0.145$

**Table 2:** Comparison between the  $F_n^{21}$  at  $t = 4M_\pi^2$ , using only S and P waves with a cutoff of 1 GeV<sup>2</sup>, calculated with data parametrizations or our IAM amplitudes.

### 3 Results

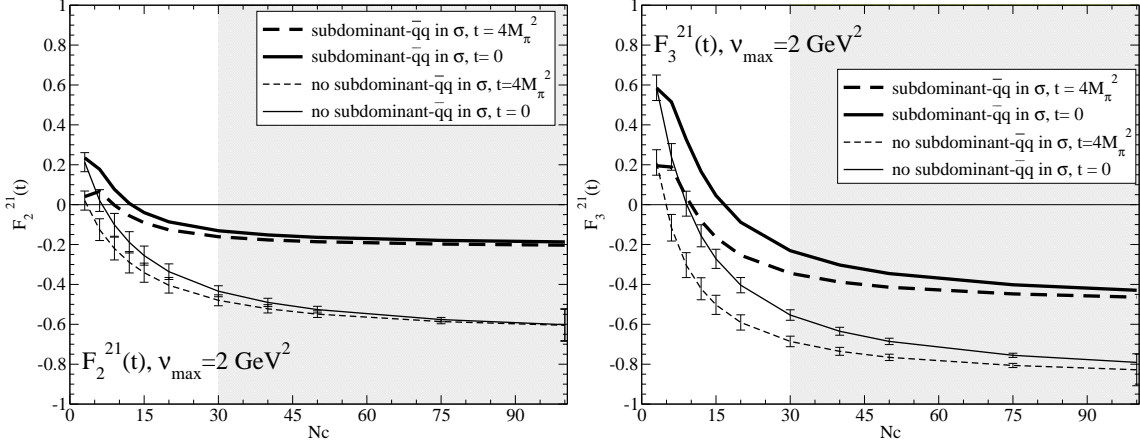
Using the IAM we can study the behaviour of Eq. (3) with  $N_c$  [1], and then check if  $|F(t)_n^{21}| \ll 1$  when increasing the number of colors. However, the IAM is only valid in the low energy region, and we have to check the influence of the high energy part on this cancellation. For this reason, in Table 1 we first calculate the value of the FESR for different cutoffs using the  $\pi\pi$  KPY [15] data parametrization as input. We thus check that local duality is satisfied for  $N_c=3$  since  $|F(t)_n^{21}| \ll 1$ , and at least for  $n=2,3$ , the main contribution to the FESR suppression occurs below 1 GeV, where we can apply the IAM. Therefore, we can use the IAM to study local duality, and check the FESR suppression with  $N_c$ . In evaluating the amplitudes  $\text{Im}A^{sI}$ , we represent these by a sum of s-channel partial waves, so that:

$$(6) \quad \text{Im} A^{sI}(s, t) = \sum_J (2J+1) \text{Im} t^{IJ}(s) P_J(\cos(\theta_s)).$$

However, using the IAM only S0, P and S2 waves can be described. It is necessary to check the effect of those waves in Eq. (3). In Table 2 we show how the influence of higher waves is around 10%, and that the IAM predicts correctly the FESR suppression.

Let us now increase  $N_c$ : if the  $\rho(770)$  mass remains constant and its width becomes narrower, but the  $f_0(600)$  contribution to the total cross section below 1 GeV becomes less and less important, then the ratios  $|F(t)_n^{21}|$  grow and there is a conflict with semi-local duality. This is shown in the thin lines of Figure 2 (Note the gray area above  $N_c = 30$ , where we consider the IAM merely qualitative). Note however, that this is a generic problem for any model where the  $f_0(600)$  contribution vanishes, not just the IAM. However if there is a subdominant  $\bar{q}q$  component for the  $f_0(600)$  with a mass somewhat above 1 GeV, as occurs

naturally within the two-loop IAM—but also in a part of the one-loop parameter space—this is enough to ensure the cancellation with the  $\rho(770)$  contribution. The effect is shown by the thick lines of Figure 2.



**Figure 2:** At  $O(p^4)$ , solid line, there is no FESR suppression and local duality fails as  $N_c$  grows. However, at  $O(p^6)$ , dashed line, the  $\sigma$  subleading  $\bar{q}q$  component ensures local duality even when increasing  $N_c$

## 4 Conclusions

The  $1/N_c$  expansion of ChPT unitarized using the IAM shows that the  $f_0(600)$  meson is not predominantly a  $\bar{q}q$  state, since the  $\sigma$  amplitude becomes smaller and smaller below 1 GeV as  $N_c$  increases. This different behavior from that of a  $q\bar{q}$  state as the  $\rho(770)$ , leads to a potential conflict with semi-local duality. However, unitarized ChPT calculations also suggest a subdominant  $\bar{q}q$  state that emerges somewhat above 1 GeV. This subdominant component ensures that semi-local duality is still satisfied as  $N_c$  increases.

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